OC3140
HW/Lab 3 Probability

1. Suppose that auto engine cylinders fire independently and fail with probability equal to 0.1. Assuming that an engine makes a successful running if at least one-half of its cylinders fire, determine whether a 4-cylinders engine or a 6-cylinder engine has the higher probability for a successful running.

Solution:

This is a Binomial Distribution (or Bernoulli Process) (see Chapter III-10),

$$P_r\left(r\right) = C_r^n \cdot p^n \left(1-p\right)^{n-r} = \frac{n!}{r!\left(n-r\right)!} p^n \left(1-p\right)^{n-r}$$

where n is the cylinder number, r is the fired cylinder number.

$q = 0.1$ and $p = 1 - q = 0.9$.

For 4-cylinder:

$$\begin{aligned} P_r\left(r \geq 2\right) &= P_r\left(r = 2\right) + P_r\left(r = 3\right) + P_r\left(r = 4\right) \\ &= C_2^4 \cdot 0.9^2 \cdot 0.1^2 + C_3^4 \cdot 0.9^3 \cdot 0.1 + C_4^4 \cdot 0.9^4 \\ &= 0.9963 \end{aligned}$$

For 6-cylinder:

$$\begin{aligned} P_r\left(r \geq 3\right) &= P_r\left(r = 3\right) + P_r\left(r = 4\right) + P_r\left(r = 5\right) + P_r\left(r = 6\right) \\ &= C_3^6 \cdot 0.9^3 \cdot 0.1^3 + C_4^6 \cdot 0.9^4 \cdot 0.1^2 + C_5^6 \cdot 0.9^5 \cdot 0.1 + C_6^6 \cdot 0.9^6 \\ &= 0.9985 \end{aligned}$$

Result: 6-cylinder engine has higher probability than 4 cylinder engine.

2. Given a normal distribution with $m = 30$ and $s = 6$. Find

   a. the normal-curl area to the right of x=17;
   b. the normal-curl area to the left of x=22;
   c. the normal-curl area between x=32 and x=41;
   d. the value of x that has 80 % of the normal-curve area to the left;

Solution:

Normal Distribution (see Chapter III, p18-19)

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right], \qquad \text{and} \qquad \mu = 30, \sigma = 6$$

Transform to the standard normal distribution as:

$$p(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad \text{where } z = \frac{x-\mu}{\sigma}$$

a. $z = \dfrac{17-30}{6} = -2.16667$,

from the standard normal distribution table (CH.3 p.23)

$$\begin{aligned} P(z > -2.16667) &= P(z < 2.16667) = 1 - P(z > 2.16667) \\ &= 1 - 0.01513 = 0.98487 \end{aligned}$$

b. $z = \dfrac{22-30}{6} = -1.3333$, $P(z < -1.3333) = P(Z > 1.3333) = 0.0912$

c. $z_1 = \dfrac{32-30}{6} = 0.3333$, $z_2 = \dfrac{41-30}{6} = 1.8333$

$$\begin{aligned} P(0.3333 < z < 1.833) &= P(z > 0.3333) - P(z > 1.833) \\ &= 0.3694 - 0.0334 = 0.336 \end{aligned}$$

d. That equal the value x has 20 % of the normal-curve area to the right.

As

$$P(z > 0.84) = 0.2005, \text{ and } P(z > 0.85) = 0.1977.$$

Use linear interpolation, it could be:

$$P(z > 0.8418) = 0.2, \text{ and } x = \mu + \sigma \cdot z = 35.05.$$

3. A company pays its employees an average wage of \$9.25 an hour with a standard deviation of 60 cents. If the wages are approximately normally distributed and paid to the nearest cent.

    a. What is the percentage of the workers receiving wages between \$8.75 and \$9.69 an hour inclusive?

    b. What is the lowest hourly wage for the highest 5 % of the employees?

Solution:

$m = 9.25$ and $s = 0.6$

a. $z_1 = \dfrac{8.75 - 9.25}{0.6} = -0.8333$ and $z_2 = \dfrac{9.69 - 9.25}{0.6} = 0.7333$

$$\begin{aligned} P(-0.8333 < z < 0.7333) &= P(z > -0.8333) - P(z > 0.7333) \\ &= P(z < 0.8333) - P(z > 0.7333) \\ &= 1 - P(z > 0.8333) - P(z > 0.7333) \\ &= 1 - 0.2024 - 0.2317 = 0.5659 \end{aligned}$$

So 56.59% of workers earn between \$8.75-\$9.69/hr.

b. From the table $P(z > 1.64) = 0.0505$ and $P(z > 1.65) = 0.0495$,

$P(z > 1.645) = 0.05$, and $x = m + s \cdot z = 9.25 + 0.6 \cdot 1.645 = 10.24$

So 5% of workers earn more than \$10.24/hr.

4. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If the average life is 780 hours, find a 96 % confidence interval for all bulbs produced by this firm.

Solution:

$m = 780$ and $s = 40$.

From the table,

$P(z > 2.05) = 0.0202$ and $P(z > 2.06) = 0.0197$, so $P(z > 2.054) = 0.02$,

$P(-2.054 < z < 2.054) = 1 - 0.02 - 0.02 = 0.96$.

$$x_1 = m + s \cdot z_1 = 780 - 40 \cdot 2.054 = 697.84$$

$$x_2 = m + s \cdot z_2 = 780 + 40 \cdot 2.054 = 862.16.$$

The life of 96% of the bulbs is between 697.84-862.16 hr.